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Abstract

We model a corporate firm in a competitive market setting, consisting of a production technology, a hierarchical organization structure, a cost efficiency parameter, and an internal pay-system. The depth of the firm determines its output and is set by profit maximization under the participation restriction that wages meet reservation wages. Reservation wages are endogenously determined in the corporate market economy. We present conditions guaranteeing a finite optimal firm size. Using CES-production technologies we illustrate how firm size depends on labor substitutability, and show that a linear technology yields the deepest organization structure, and complementarity between workers yields the flattest structure.

JEL-classification: *D23, J24, L22.*
Keywords: *Hierarchy, cooperative production, optimal firm size, positional wages.*

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1 Introduction

This paper addresses the entrepreneurial problem of determining endogenously the optimal size of a firm in a competitive market environment, when the production possibilities interact with the labor organization and the governance of the firm. It has been observed by Rajan and Zingales (2001) that at the root of most enterprizes generating economic surplus is an entrepreneur with a unique critical resource such as an idea, good customer relationships, a new tool, or superior management technique. We represent this unique resource by two concepts: a production outcome function specifying the various production possibilities from which one to select, and the firm's organization, specifying the management of transactions and agency relations in the firm when realizing the chosen production technology. The firm's organization – to be determined by the entrepreneur – specifies roles or positions within the firm and the agency relations connecting these roles and positions. Given the pay-system and the market prices, the entrepreneur adapts the organization to the technology and so determines the size of the firm. The interaction between technology and organization determines the productivity of the workers' positions in the organization. Such a firm is called a 'corporate firm'.

We follow the seminal paper of Alchian and Demsetz (1972) who state that production is in principle a collective effort, see also Hart and Moore (1990) and Ichiishi (1993). Therefore we use a cooperative game theoretic model in which teams of workers can generate a specific level of production when they are coordinated by their respective managers. The technology is a set of production outcome functions, defined on the set of front-workers in the organization for each level of production. The chosen level of production determines the size of the firm.

The firm's organization is described by a network of principal-agent relations between the labor positions in the firm. The top-position or the CEO of the firm – having no principal in the firm – chooses the structure and the size of the organization such that it maximizes profit given external prices and an internal pay-system. The network of principal-agent relations is a hierarchy represented by a tree. The front-positions in the firm – having no agent as successor in the firm – interact with customers and generate the added value of the firm. These front-positions enter the firm's production function as inputs. The positions between the top and the front workers, the middle-management, facilitate the productivity of the front-workers. Each middle-manager or coordinator is a principal for some other agents at a lower level, and translates, monitors, and adapts the various tasks and responsibilities of its subordinates in order to let them comply with the overall mission of the firm. This mission is given by the CEO or top-principal, who aims at profit maximization[1]. Increasing the number of levels broadens the productive base of the firm but also increases the level-

[1]Although not written down explicitly here, this mission may be reformulated to subgoals down in the firm organization and how, in the upward direction, information about production and sales is communicated and translated in operational terms for higher levels in the firm organization. See Radner (1992) for a survey about the information processing and decentralized decision making in hierarchical firms.

dependent agency costs. These costs are increasing in the number of hierarchical levels. Examples of such costs are the translation of the central strategic mission to each operational level, or the agency costs involved in the processing and control of level-dependent budgets and information, implying a loss of control of a coordinator over the behavior of its direct subordinates, see, e.g. Williamson (1967). These agency costs are expressed by a discount factor between 0 and 1, which indicates the percentage loss of output for each level involved.

The exogenously given pay-system assigns payments to all positions in the firm. It is a function distributing the value added of the firm between the profit for the owner position and the different wages for the employee positions in the firm. We use a cooperative distribution function which implies that all positions earn a share in the value added that can be generated by different compositions of actively occupied positions in the firm. This distribution function satisfies three axioms that resemble conditions used in collective wage agreements. Since the wages and profit are assigned to the positions in the firm, and not directly to the employees who occupy these positions, we call them *positional wages* and *positional profit*. The endogenous determination of these positional wages is a novelty that distinguishes our approach from the models that follow the seminal papers of, for example, Williamson (1967) and Keren and Levhari (1979, 1983), where the wages of the workers are fixed and independent of the firm structure.

On the labor market, potential employees are offered positional wages for the various positions in the firm. If an employee accepts an offer, he is admitted to a position in the organization and voluntarily subjects himself to the hierarchy of the firm, see, e.g., Coleman (1980), Rosen (1982), and Simon (1991). If some positional wage falls below the reservation wage that position will not be occupied. The goal of the firm's CEO is to maximize profit given the workers' participation constraint. If maximal profit falls below the external reservation profit then the owner will not activate the firm. In principle, the profit maximizing owner may be forced to pay a position wage to an employee higher than the market wage if the pay-system assigns that wage according to his or her internal productivity[2].

In order to endogenize the reservation wages and reservation profit, we introduce a general equilibrium framework, called a *corporate market economy*, which consists of a representative firm as described above, a finite set of consumers and a finite set of competitive markets[3]. The firm demands labor and supplies the goods or services produced, while the consumers demand these goods or services and supply labor. So wages and prices interact with the optimal size of firms. The firm's supply and demand is determined by the optimal level of the organization, as set by the entrepreneur. Market supply of the consumption good and

[2]The idea that wages may be higher than reservation wages also appears in *efficiency wage theory*, see e.g., Stiglitz (1976), Akerlof (1984) and Yellen (1984).

[3]This context implies that the corporate firm's outcome is marketable. The corporate firm can also be interpreted as a non-profit or public firm. In that case, the general equilibrium framework will be much more complex.

market demand for labor then is determined by assuming a representative firm. Assuming consumer preferences to satisfy the standard regularity conditions, aggregation of individual demand and supply yields market demand for the consumption good and market supply of labor. A corporate market equilibrium consists of an output price, a wage and a firm size, such that (i) firm size is optimal given the prices, and (ii) the prices are competitive equilibrium prices at which market supply equals market demand given the firm size. In other words, in a corporate market equilibrium the external competitive equilibrium prices are consistent with the firms' internal equilibrium (i.e. profit maximizing size). In general there may arise discontinuities in the firms supply and demand functions. As a consequence, a corporate market equilibrium need not exist. However, such an equilibrium exists if the optimal firm size is continuous in prices. This is the case for certain production technologies, such as linear and Cobb-Douglas technologies. We show the effect of a change in technology on the firm's organization and on positional wages.

The paper is organized as follows. In Section 2, the model of a corporate firm is introduced. In Section 3 we provide conditions under which an optimal firm size exists, i.e. when its level is finite. In Section 4 we discuss some special cases using constant elasticity of substitution (CES) production technologies. We show that in case working labor is substitutable (i.e. there are no complementarities), for reasonable values of the effectiveness parameters the owner will choose the deepest organization structure restricted by the reservation wage of workers. An opposite result is derived for Cobb-Douglas production technologies with indispensable labor, in which case the flattest organization is chosen. In Section 5 we discuss the feature that the optimal firm size might depend discontinuously on the market prices, and discuss the consequences for a corporate market economy, in particular with respect to non-existence of equilibria. Finally, in Section 6 we describe related literature and give some concluding remarks.

2 The corporate firm

In this section we introduce the constitutional elements of the firm: the technology, the internal organization, the agency costs and the pay-system.

2.1 The technology: outcome possibilities

For every level n of the organization, the set of employees of a firm, N_n, is partitioned in a set of front-workers, W_n, and a set of managers or coordinators, M_n (including the unique owner or CEO). The firm's technology is described by a set of (production) outcome functions defined on the power set of front-workers, $f_n\colon 2^{W_n} \to \mathbb{R}$, which set follows from the decision made by the owner. This set contains all coalitions of front-workers, where a coalition is defined as a group of workers who interact only with members of their group. So, for some activity all external effects are internalized in that coalition. In order to simplify the analysis, we assume that all front-workers are homogeneous meaning that they have identical roles in the production process. So, the firm's outcome function can be reduced to $f_n\colon \{1, \ldots, |W_n|\} \to \mathbb{R}$, defined on the number of (identical) front-workers. We further assume that the outcome function is *monotone* implying that $f(k) \leq f(l)$ if $k \leq l$. An important subclass of monotone outcome functions is the class of supermodular outcome functions (see. e.g. Milgrom and Roberts (1994)) which exhibit increasing scale returns in the sense that they favor producing with larger sets of front-workers. Moreover, we assume that nothing is produced if no worker is providing any labor input, i.e., $f(0) = 0$.

As mentioned above, we parameterize the size of a firm by the variable n. The exact meaning of this variable is to be specified in the next subsection. So, the set of outcome functions at the disposal of the entrepreneur is defined by $\{f_n \mid n = 1, \ldots, \overline{n}\}$, with $\overline{n}$ sufficiently large. The outcome provided by the firm is sold at a competitive output price $p > 0$. Thus, if all front-workers are active, then the *transaction value* is the firm's outcome multiplied by the (market) price and is equal to $pf_n(|W_n|)$.

2.2 The internal organization of the firm

A service cannot be rendered by the front-workers unless it is managed and facilitated by the middle-management. The main tool of management for the entrepreneur is the internal organization structure of the firm. It may be seen as the institutional hull that realizes and supports, but also constraints the technology, represented here by the set of outcome functions. The organization of a firm determines tasks, competencies, and incentives for the various roles in a firm such that the expected performance of the middle management optimally supports the productivity of the front-workers, in order to maximize the firm's value-added.

For any given value of the level n, the firm's governance is described by a *hierarchical network* (N_n, S_n) of principal-agent relations, in which the set of nodes, N_n, represents a set of well defined roles[4] or labor position in the firm, and the structure $S_n\colon N_n \to N_n$ represents the set of principal-agent relations, in short *agency relations*, where (i, j) with $j \in S_n(i)$, is an agency relation with principal i and agent j if There is a unique position having no principal, called the *top position*, i_0, which will be occupied by the owner or CEO of the firm. Each agent has one principal, so there is no cycle in the graph. It follows that the internal organization structure (N_n, S_n) has a tree structure, its root being the top-position i_0, and the end-points forming a nonempty set of positions having no agents. This set corresponds with the set of *front-positions* in the firm: $W_n = \{i \in N_n \mid S_n(i) = \emptyset\}$. The labor input of the production process is provided by the workers occupying the front-positions. Obviously,

[4]The model is therefore a role assignment model as introduced by Everett and Borgatti (1991), see also Pekec and Roberts (2001).

a firm with structure (N_n, S_n) can produce according to a production outcome function f_n if and only if the set of front-positions in the structure W_n corresponds to the domain of the outcome function. The other positions, i.e. the ones in the set $M_n = N_n \setminus W_n$, are the *coordinator positions* (including the top-position i_0) which serve to increase the productivity of the front-positions.

Since the internal organization structure has a hierarchical tree structure, different *echelons* can be distinguished in the internal organization, where each position in a given echelon has the same distance to the top-position. Let $N_0 = \{i_0\}$ represent the top-echelon with the owner-position of the firm. Then, recursively we define the sets $N_\ell = N_{\ell-1} \cup \{i \in N_n \setminus N_{\ell-1} \mid i \in S_n(j)$ for some $j \in N_{\ell-1}\}$, for $\ell = 1, \ldots, n$. So, the sets $L_\ell = N_\ell \setminus N_{\ell-1}$, $\ell = 1, \ldots, n$, form the different hierarchical echelons in the firm. Additional structure is required to guarantee that the lowest echelon $N_n \setminus N_{n-1}$ is equal to the set of positions having no agent. This is obtained by assuming that each principal in the firm has the same number of agents[5]. This number is called the *span of control*[6] and is denoted by s. So $|S_n(i)| = s$ for all $i \in M_n = N_n \setminus W_n$. Given the top-echelon $L_0 = N_0$, the positions at some echelon, ℓ, of the firm are represented by $L_\ell = \{i_{\ell,1}, \ldots, i_{\ell,s^\ell}\}$, for $\ell = 1, \ldots, n$. Denoting the top-position i_0 alternatively by $i_{0,1}$ as the first position in echelon 0, the corresponding relational structure is $S_n(i_{l,k}) = \{i_{l+1,(k-1)s+1}, \ldots, i_{l+1,ks}\}$, $l = 0, \ldots, n-1$ and $k = 1, \ldots, s^l$. We refer to the number of echelons n as the *level* of the firm. In Figure 1 the internal organization structure of a one-level and two-level firm is illustrated for the case that the span of control, s, equals 2.

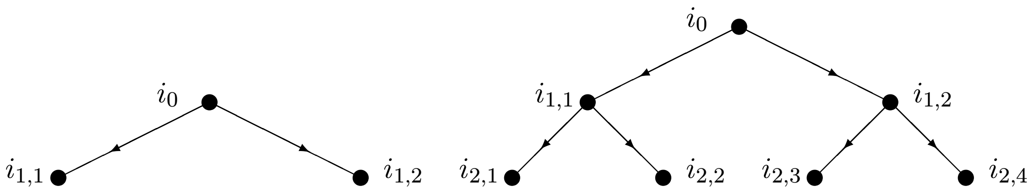

Figure 1: A one-level and a two-level internal organization structure with span of control 2

In an n-level firm the number of positions equals $|N_n| = \sum_{\ell=0}^{n} s^\ell = \frac{(s^{n+1}-1)}{(s-1)}$, the number of coordinator positions equals $|M_n| = \sum_{\ell=0}^{n-1} s^\ell = \frac{s^n-1}{s-1}$, and the number of front-positions

[5] We can allow for firms with a non-uniform span of control and front-positions in different echelons. However for notational convenience we do not allow for such firms in this paper.

[6] Since the workers have to be coordinated, their number is not arbitrary and depends on the size of the internal organization structure. The simplifying assumption that the span of control is equal for each coordinator can only be made if the workers are homogeneous and, for each echelon in the firm, the coordinators are identical. Between echelons their capacities and tasks will differ. In such a firm the number of echelons completely determines the structure of the firm.

is equal to $|W_n| = s^n$. The *coordination intensity* [7] equals the ratio $|M_n|/|W_n|$ and is approximately equal to $1/(s-1)$, if n is large enough.

2.3 Agency costs

The sequence of agency relations decentralizes decision making at each consecutive echelon and allows to decrease the complexity of the decision problem at each echelon. It results, however, in certain level-dependent agency costs. These *agency costs* per echelon are stated as a percentage of final production and are represented by $(1-\alpha)$, with the parameter α strictly between zero and one. Agency costs are therefore increasing in the level of the firm. Increasing the level (i.e. adding an echelon) in the firm structure may thus benefit the owner by increasing the scale of production, at the cost of an increase in agency costs.

The owner of the firm gives the workers access to the production technology by allowing the workers to occupy these positions. As we have seen, if all positions are effectively occupied then a transaction value equal to $pf_n(|W_n|) = pf_n(s^n)$ is generated. Net revenue or *value added* is obtained by subtracting the level-dependent cost from this gross revenue[8] yielding $p\alpha^n f_n(s^n)$. Note that the parameter α can be seen as an *agency efficiency* parameter. It may correlate with the span of control parameter s, but both are given here.

2.4 The pay-system

The pay-system distributes the value added of a firm among the worker and owner positions. We assume that all capital costs are incurred by the owner. The pay-system must be equally applicable to any organization of a firm. So, the *pay-system* is a function φ which assigns a distribution of value added to every cooperative outcome function f_n with internal organization structure (N_n, S_n) and level-dependent agency efficiency parameter α. Since the internal organization structure and level dependent agency efficiency are determined by the level n, we denote the reward assigned to position $i \in N_n$ in a firm producing according to f_n by $\varphi_i(f_n)$. This pay-system determines the wages that eventually are paid to the employees occupying the coordinator and front-positions. Since the rewards are assigned to and depend on the positions in the firm structure we refer to these wages as *positional wages*. Similarly, we refer to the profit as *positional profit*.

We assume this pay-system to satisfy the following three properties. First, it guarantees a balanced budget[9], i.e., $\sum_{i \in N_n} \varphi_i(f_n) = p\alpha^n f_n(s^n)$ for every firm level n and outcome function f_n. Second, it satisfies *vertical monotonicity* meaning that a supervisor does not receive a lower wage than its successors, i.e., for every firm level n and every monotone

[7] This concept is introduced by Gordon (1994) in order to estimate and explain differences in the span of control over various countries.

[8] For notational convenience we do not consider material cost that depend on the level of production. Considering these costs to have given input price $c > 0$ does not change the results.

[9] In game theory this property is called *efficiency*.

outcome function f_n, it holds that $\varphi_i(f_n) \geq \varphi_j(f_n)$ for all $i \in M_n$ and $j \in S_n(i)$. Finally, we assume the pay-system to be *symmetric* meaning that in a homogeneous firm it assigns the same positional wage to all positions within the same coordination or worker level. For a homogeneous firm this last property implies that we can speak about wages assigned to echelons instead of wages assigned to positions, i.e., $\varphi_k(f_n) = \varphi_i(f_n)$ for all $i \in N_k$, $k \in \{1, \ldots, n\}$. Similarly, the profit of the owner position is denoted by $\varphi_0(f_n)$.

Now we can define an **n-level corporate firm** as a tuple $F_n = (f_n, (N_n, S_n), \alpha, \varphi)$ with:

1. a monotone outcome function f_n defined on a set W_n,
2. an internal organization structure (N_n, S_n) having a tree structure with a constant span of control s and level n on the non-empty and finite set of positions N_n such that the set of front-positions $\{i \in N_n \mid S_n(i) = \emptyset\}$ corresponds to W_n,
3. an agency efficiency parameter $0 < \alpha < 1$,
4. a pay-system φ that yields a distribution of value added in positional wages and profit satisfying budget neutrality, vertical monotonicity and symmetry.

In order to endogenously determine the optimal firm level we need to define the corporate firm with a variable number of levels, i.e. we need to describe how the firm looks like for every possible size.

Definition 2.1 *A* **corporate firm** *is a set $F = \{F_n\}_{n \in \mathbb{N}}$ with $F_n = (f_n, (N_n, S_n), \alpha, \varphi)$ being an n-level corporate firm for every $n \in \mathbb{N}$.*

3 The optimal size of the corporate firm

Since the profit of a corporate firm depends on the firm level, the owner can determine profit by choosing this level. Given the span of control, the only way to increase the number of front-positions in the firm is to increase its level. An extra echelon allows more workers to be active. It has a positive effect on value added through the monotone outcome function. On the other hand, there is the negative effect of the level dependent agency cost. The owner of the firm chooses level n in order to maximize profit.

Suppose that the owner can choose a firm level between 1 and n without any constraints. In that case the maximal profit equals $\overline{\varphi}_0(n) = \max\{\varphi_0(f_{\overline{n}}) \mid \overline{n} \in \{1, \ldots, n\}\}$ with $\overline{\varphi}_0(0) = 0$. Thus, by $\overline{\varphi}_0$ we have written profit as a non-decreasing function of maximal possible firm level n. We call $\overline{\varphi}_0$ the *level-dependent profit function*, see Figure 2. (In the figures we assume for simplicity that the level n can be any non-negative real number $n \in \mathbb{R}_+$.)

Besides the unit output price $p > 0$, the *external* organization of the firm is represented by a *reservation wage* $w > 0$ for workers, and a *reservation profit* $\pi > 0$ for the owner. In order for the firm to be active, the front- and coordinator positions have to be occupied

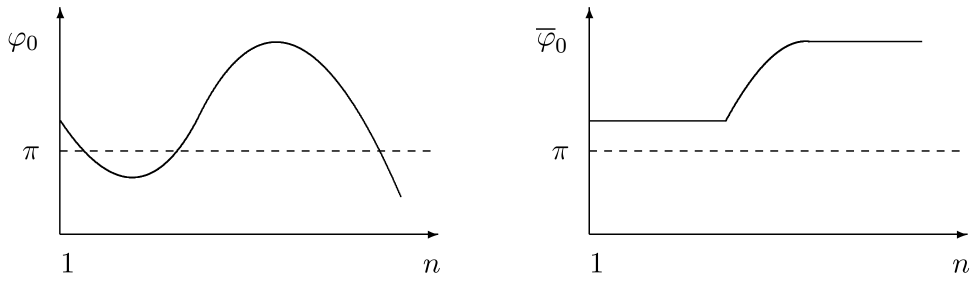

Figure 2: Profit φ_0 and level dependent-profit $\overline{\varphi}_0$ for the owner as function of firm level

by employees (i.e. front-workers and coordinators). The potential employees of a firm will accept a position in a firm with level n if and only if the positional wages offered do not fall below their reservation wage w. Therefore, in optimum the owner chooses firm level n such that profit is maximal under the constraint that the positional wages offered to the workers[10] do not fall below their reservation wage. If profit at this level is lower than the reservation profit, then the owner will not activate the firm. So, the only firm levels that are *supported by the external environment of the firm* are the ones in the set $N(w, \pi, p) = N^o(w, p) \cap N_o(\pi, p)$, where $N^o(w, p) = \{n \in \mathbb{N} \mid \varphi_n(f_n) \geq w\}$ is the set of levels that satisfy the *worker participation constraint*, and $N_o(\pi, p) = \{n \in \mathbb{N} \mid \varphi_0(f_n) \geq \pi\}$ is the set of levels that satisfy the *owner participation constraint*. In general, the set $N(w, \pi, p)$ can be empty, disconnected or unbounded. The optimal coordination level n^* is the lowest level of coordination that maximizes profit under the worker and owner participation constraints.

Definition 3.1 *The* **optimal firm level** *of a corporate firm F, given reservation wage w, reservation profit π and output price p, is the level*

$$n^*(w, \pi, p) = \min\{n \in N(w, \pi, p) \mid \varphi_0(f_n) = \max_{\hat{n} \in N(w,\pi,p)} \varphi_0(f_{\hat{n}})\},$$

if there exists $n \in N(w, \pi, p)$ with $\varphi_0(f_n) \geq 0$, and $n^*(w, \pi, p) = 0$ otherwise.

The following proposition gives sufficient conditions for the existence of a finite optimal firm level. We say that average labor productivity is non-increasing in firm level if there is a level $\tilde{n}$ such that $f_{n+1}(s^{n+1}) \leq s f_n(s^n)$ for all $n \geq \tilde{n}$.[11]

[10]We implicitly assume that the labor market is compartmentalized into homogeneous levels and is sufficiently differentiated to provide for each hierarchical level a competitive partial labor market. Any level may be chosen as a benchmark. We have chosen the lowest level. This assumption allows us to compare only the lowest wage offered by the firm with the reservation wage of the corresponding compartment on the labor market since vertical monotonicity of the pay-system implies that the wage offered to a coordinator is always greater or equal to the wages offered to its subordinate workers. Thus, if the workers accept the wages offered then also the coordinators accept the wages offered to them.

[11]The more strict property of strongly non-increasing average labor productivity requires that this inequality holds for all $n \geq 1$. For our result this is not necessary.

Proposition 3.2 (Existence of a finite optimal firm level) *For every corporate firm F with non-increasing average labor productivity in firm level, and for every triple of prices (w, π, p), the set $N(w, \pi, p)$ of firm levels that are supported by the external environment is bounded, and thus a finite optimal firm level exists.*

Proof
First, suppose that average labor productivity is non-increasing in firm level $n \geq 0$. Then there exists a constant $c \in \mathbb{R}_+$ such that $f_n(s^n) \leq cs^n$, and thus total value added of a firm with level n satisfies $p\alpha^n f_n(s^n) \leq pc(\alpha s)^n$. Since the number of positions in an n level firm equals $|N_n| = \frac{s^{n+1}-1}{s-1}$, efficiency, vertical monotonicity and symmetry of the pay-system φ imply that $\varphi_n(f_n) \leq \frac{p\alpha^n f_n(s^n)}{|N_n|} \leq \frac{pc(\alpha s)^n(s-1)}{s^{n+1}-1} = \frac{pc\alpha^n(s^{n+1}-s^n)}{s^{n+1}-1} \leq pc\alpha^n$. So, for $\alpha < 1$ it holds that $\lim_{n\to\infty} \varphi_n(f_n) = 0$. But then $\{n \in \mathbb{N} \mid \varphi_n(f_n) \geq w\}$ is bounded for $w > 0$, and so is $N(w, \pi, p) \subset \{n \in \mathbb{N} \mid \varphi_n(f_n) \geq w\}$.

Next, suppose that there exists a $\tilde{n} \in \mathbb{N}$ such that average labor productivity is non-increasing in firm size $n \geq \tilde{n}$. Then there exists a constant $c \in \mathbb{R}_+$ such that for $n \geq \tilde{n}$, it holds that $f_n(s^n) - f_{\tilde{n}}(s^{\tilde{n}}) \leq cs^{n-\tilde{n}}$, and thus for $\alpha < 1$ we have $p\alpha^n f_n(s^n) - p\alpha^{\tilde{n}} f_{\tilde{n}}(s^{\tilde{n}}) \leq p\alpha^n(f_n(s^n) - f_{\tilde{n}}(s^{\tilde{n}})) \leq p\alpha^n cs^{n-\tilde{n}} = \frac{pc(\alpha s)^n}{s^{\tilde{n}}}$. Then $\varphi_n(f_n) \leq \frac{p\alpha^n f_n(s^n)}{|N_n|} \leq \frac{p\alpha^{\tilde{n}} f_{\tilde{n}}(s^{\tilde{n}})(s-1)}{s^{n+1}-1} + \frac{(p\alpha^n f_n(s^n) - p\alpha^{\tilde{n}} f_{\tilde{n}}(s^{\tilde{n}})(s-1)}{s^{n+1}-1} \leq \frac{p\alpha^{\tilde{n}} f_{\tilde{n}}(s^{\tilde{n}})(s-1)}{s^{n+1}-1} + \frac{pc\alpha^n(s^{n+1}-s^n)}{s^{\tilde{n}}(s^{n+1}-1)} \leq \frac{p\alpha^{\tilde{n}} f_{\tilde{n}}(s^{\tilde{n}})(s-1)}{s^{n+1}-1} + \frac{pc\alpha^n}{s^{\tilde{n}}}$. So, for $\alpha < 1$ it holds that $\lim_{n\to\infty} \varphi_n(f_n) = 0$, and thus $\{n \in \mathbb{N} \mid \varphi_n(f_n) \geq w\}$ is bounded for $w > 0$, and so is $N(w, \pi, p)$.

This immediately yields that the optimal firm level is finite. □

Note that without a finite optimal firm level our model would not be suitable. In Proposition 3.2 we stated conditions under which the set $N(w, \pi, p)$ is bounded. However, it can still be disconnected or empty (see Figures 3 and 4).

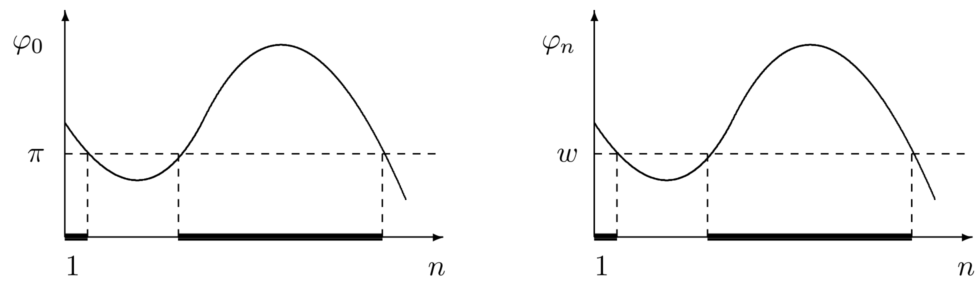

Figure 3: $N(w, \pi, p) \neq \emptyset$ but disconnected.

4 The effect of technological change on firm level

We have simplified the corporate firm by assuming that the production outcome function is defined on a homogeneous set of front-workers. That still leaves room for a broad scope of

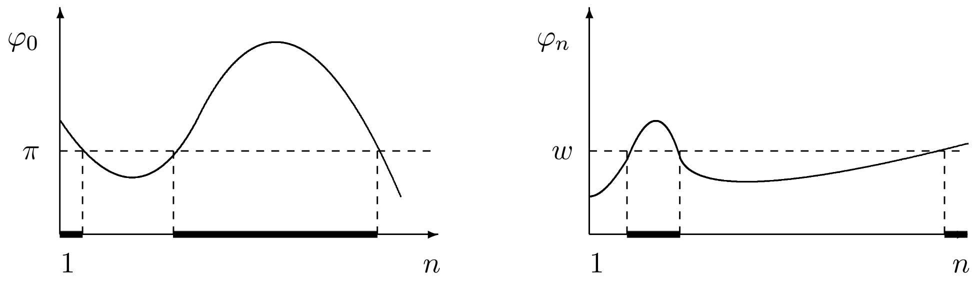

Figure 4: $N(w, \pi, p) = \emptyset$.

technologies and allows us to analyze the effects of a change in technology. We start with a linear technology in which all front-workers are substitutes, and thus there are no complementarities between labor inputs. Then a Cobb-Douglas technology is analyzed, implying complementarity between individual front-workers. Finally, we discuss the more general case of constant elasticity of substitution (CES) production technologies. In order to analyze these cases, we have to specify the pay-system for the corporate firm.

4.1 The position pay-system

A particular specification of the pay-system is the *position pay-system*[12], which distributes the value added of the corporate firm among all positions in the organization. According to this position pay-system the distribution of value added among the owner (as profit) and the employee positions (as wages) depends on the value added that can be generated by all subsets of front-positions $E \subset W_n$, i.e., all values $v^{f_n}(E) = p\alpha^n f_n(|E|)$ for $E \subset N_n$.

Given these values, we define the *dividends* of subsets of front-positions, recursively, by $\Delta^{f_n}(E) = v^{f_n}(E)$ if $|E| = 1$, and $\Delta^{f_n}(E) = v^{f_n}(E) - \sum_{\substack{F \subset E \\ F \neq E}} \Delta^{f_n}(F)$, otherwise. These dividends thus can be seen as the net-productivity of the subsets $E \subset W_n$, i.e. the dividend $\Delta^{f_n}(E)$ represents the contribution to value added that is generated by E and was not already generated by the subsets of E. Note that these dividends can be negative, even for supermodular outcome functions. For a discussion of these dividends we refer to Harsanyi (1959).

Now, the position pay-system $\widetilde{\varphi}$ distributes the dividend of a set of front-positions E equally among the front-positions in E and their *superior* positions[13]. Motivations for using

[12] Game theoretically, this position pay-system is obtained by applying the *permission value* as developed by Gilles, Owen and van den Brink (1992), van den Brink and Gilles (1996) and van den Brink (1997) to a restricted production game as done in van den Brink (1996). As such it is related to the *Shapley value* for cooperative TU-games and the *Nash Bargaining solution* for bargaining problems. We remark that it is not based on the *position value* for games with limited communication as introduced in Borm, Owen and Tijs (1992).

[13] Formally, the payments to the firm positions are given by $\widetilde{\varphi}_i(f_n) = \sum_{\substack{E \subset W_n \\ \widehat{S}_n(i) \cap E \neq \emptyset}} \frac{\Delta^{f_n}(E)}{|\widehat{S}_n^{-1}(E)|}$, for all $i \in N_n$, where for every $i \in N_n$ we have $j \in \widehat{S}_n(i)$ if and only if $i = j$ or there exists a sequence of positions $(h_1, \ldots, h_t)$ such that $h_1 = i$, $h_{k+1} \in S(h_k)$ for all $1 \leq k \leq t-1$ and $h_t = j$.

the position pay-system can be found in van den Brink (1996). Note that the position pay-system is a consistent system in the sense that it can be applied to any firm irrespective of the specific productivity of the workers, and thus it can be seen as a standard in collective wage agreements.

It is easy to verify that the position pay-system satisfies symmetry, and thus the wages of the worker positions at level n can be indicated by $\widetilde{\varphi}_n(f_n) = \widetilde{\varphi}_i(f_n)$, for all $i \in W_n$. The position pay-system also satisfies budget neutrality and structural monotonicity, so Proposition 3.2 is valid for $\widetilde{\varphi}$.

In the previous section we also mentioned that the set $N(w, \pi, p)$ of firm levels that are supported by the external environment of a firm can be empty. For firms with supermodular outcome functions and the position pay-system, sufficient conditions for the existence of a positive optimal level in $N(w, \pi, p)$ follow from the following proposition.

Proposition 4.1 *Let F be a corporate firm with the position pay-system $\widetilde{\varphi}$ and supermodular outcome functions $\{f_n \mid n \in \mathbb{N}\}$. If productivity meets reservation prices in the sense that $\frac{v^{f_1}(W_1)}{2s} \geq w$ and $\frac{v^{f_1}(W_1)}{s+1} \geq \pi$ for every $w, \pi, p > 0$, then the set $N(w, \pi, p)$ is nonempty.*

Proof

In van den Brink (1996) it is shown that for supermodular outcome functions with identical workers it holds that $1 \leq \frac{\widetilde{\varphi}_i(f_n)}{\widetilde{\varphi}_j(f_n)} \leq s$ for $i \in M_n,\ j \in S_n(i),\ n \in \mathbb{N}$. Applying this result to such a firm with size $n = 1$ yields

(i) $\widetilde{\varphi}_1(f_1) = \frac{v^{f_1}(W_1) - \widetilde{\varphi}_0(f_1)}{s} \geq \frac{v^{f_1}(W_1) - s\widetilde{\varphi}_1(f_1)}{s} \geq \frac{v^{f_1}(W_1)}{2s}$, and

(ii) $\widetilde{\varphi}_0(f_1) = v^{f_1}(W_1) - s\widetilde{\varphi}_n(f_1) \geq v^{f_1}(W_1) - s\widetilde{\varphi}_0(f_1) \geq \frac{v^{f_1}(W_1)}{s+1}$.

□

It follows that under some reasonable conditions the position pay-system allows the owner to activate the corporate firm.

4.2 Degrees in complementarity between front-workers

In this section we consider firms with a specific production technology. We first consider two types of well-known and widely applied constant elasticity of substitution (CES) production technologies, namely linear and Cobb-Douglas technologies.

4.2.1 Linear outcome functions: no complementarity

We first consider a linear production technology with outcome function $f_n^1(k) = k$. In this case, which coincides with Williamson (1967), the labor inputs are perfect substitutes. Although for firms with different size the domain of the outcome function is different, the production technology is the same for all levels of coordination, i.e. the firm produces according to a linear production technology for every size n. The value added of a firm with front-positions $E \subset W_n$ occupied then is given by

$$v^{f_n^1}(E) = p\alpha^n |E| \text{ for all } E \subset W_n,$$

with dividends equal to $\Delta^{v^{f_n^1}}(E) = p\alpha^n$ if $|E| = 1$, and $\Delta^{v^{f_n^1}}(E) = 0$ otherwise. Profit according to the position pay-system equals

$$\widetilde{\varphi}_0(f_n^1) = \frac{p(\alpha s)^n}{n+1}. \tag{1}$$

The workers will only accept the positions offered to them if the corresponding position wages exceed their reservation wages. The wage assigned to front-position $i \in W_n$ according to the position pay-system equals

$$\widetilde{\varphi}_n(f_n^1) = \frac{p\alpha^n}{n+1}. \tag{2}$$

Assume for simplicity that the level n can be any non-negative real number $n \in \mathbb{R}_+$. The wage assigned to the front-positions is decreasing with the firm level n (see the right picture in Figure 5)[14].

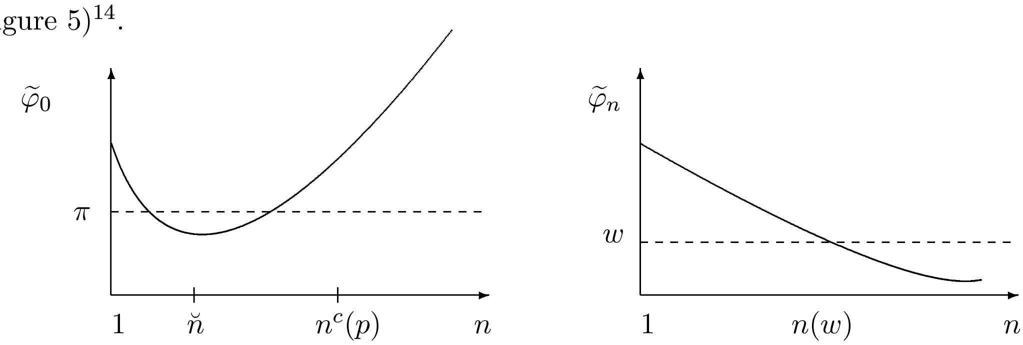

Figure 5: Profit and wages for a linear production technology.

For the moment we ignore the owner participation constraint. Clearly, in this case the set $N(w, 0, p) = [0, n(w)]$ is connected and bounded from above by the *reservation wage level* $n(w)$ defined as the firm level above which the wage of workers is lower than their reservation wage, i.e.,

$$n(w) = \max\{n \in \mathbb{R}_+ | \widetilde{\varphi}_n(f_n) \geq w\} = \max\left\{n \in \mathbb{R}_+ \left| \frac{p\alpha^n}{n+1} \geq w \right.\right\}. \tag{3}$$

The optimal level of the firm is determined by maximizing profit (see equation (1)) of the owner. It turns out that profit decreases to a minimum, attained at

$$\breve{n} = \frac{1}{\ln(\alpha s)} - 1, \tag{4}$$

[14]Since $\ln(\alpha) < 0$ (because $\alpha < 1$) it holds that $\frac{d\ \widetilde{\varphi}_n(f_n^1)}{d\ n} = \frac{p\alpha^n((n+1)\ln(\alpha)-1)}{(n+1)^2} < 0$.

and then increases monotonically (see the left picture in Figure 5)[15]. This implies that from the *critical* level $n^c(p)$, being the minimal size for which profit is at least as high as profit for a firm of level 1, there is no limit to the firm level from the point of view of the owner, and thus the owner will try to expand the firm level as high as possible due to its profit maximizing behavior. The owner, however, is restricted in this ambition by the labor market, i.e., the optimal firm level n^* should be an element of $N(w, 0, p) = [0, n(w)]$.

Next we discuss how the optimal level n^* looks like. If profit at the reservation wage level $n(w)$ exceeds profit at level one (i.e. if $\widetilde{\varphi}_0(f^1_{n(w)}) > \widetilde{\varphi}_0(f^1_1)$) then the reservation wage level $n(w)$ exceeds the critical level $n^c(w, p)$. In that case the owner will set the level of the firm equal to the reservation level $n(w)$ and, implicitly, will reduce the wage of the workers as close to their reservation wage as possible. Thus, it are the workers who determine the level of the firm.

However, if profit at the reservation wage level $n(w)$ does not exceed profit at level one (i.e. if $\widetilde{\varphi}_0(f^1_{n(w)}) \leq \widetilde{\varphi}_0(f^1_1)$), and thus $n(w) \leq n^c(w, p)$, then coordination is not profitable. If wages at level one are at least equal to the reservation wage (i.e. if $\widetilde{\varphi}_n(f^1_1) \geq w$) then the firm will have only one hierarchical level with no intermediate coordinators. Otherwise, if $\widetilde{\varphi}_n(f^1_1) < w$, the firm will not be active.

Above we ignored the participation constraint of the owner. Also considering that constraint, the above is still valid as long as $\max\{\widetilde{\varphi}_0(f^1_1), \widetilde{\varphi}_0(f^1_{n(w)})\} \geq \pi$. Of course, $n = 0$ if $\max\{\widetilde{\varphi}_0(f^1_1), \widetilde{\varphi}_0(f^1_{n(w)})\} < \pi$.

Note that $\breve{n} = \frac{1}{\ln(\alpha s)} - 1 \leq 1$ if and only if $\alpha s \geq e^{\frac{1}{2}} \simeq 1.65$. Thus, if $\alpha s \geq e^{\frac{1}{2}} \simeq 1.65$ then profit of the owner attains its minimum at $\breve{n} \leq 1$ and profits are monotonically increasing in $n \geq 1$. In this case the owner will always push the workers to their reservation wages, and set the level of the firm equal to $n(w)$. For $s = 2$ this means that $\widetilde{\varphi}_0$ is increasing in n for $\alpha \geq \frac{1}{2}e^{\frac{1}{2}} \simeq 0.825$. For $s > 2$ this is even the case for lower values of α. Williamson (1967) argues that α mostly will be in the neighborhood of 0.9. So, we might expect profit to be increasing in n, and the level of the firm to be determined by the reservation wages of the workers.

Proposition 4.2 (Linear outcome functions) *Consider a firm with a linear production technology and the position pay-system. If an optimal firm level exists and $\alpha \geq \frac{1}{2}e^{\frac{1}{2}}$, then profit is monotonically increasing in firm level $n \geq 1$, and the owner will choose the* **deepest** *organization structure $n^* = n(w)$, which level is bounded above by the reservation wage of the workers.*

[15] The first order condition for profit maximization yields $\frac{d\ \widetilde{\varphi}_0(f^1_n)}{d\ n} = \frac{p(\alpha s)^n((n+1)\ln(\alpha s)-1)}{(n+1)^2} = 0$, so $n = \frac{1}{\ln(\alpha s)} - 1$. It may be noticed that $\alpha s > 0$, otherwise there would be no reason to expand. For $n = \frac{1}{\ln(\alpha s)} - 1$, the second order condition yields $\left.\frac{d^2\ \widetilde{\varphi}_0(f^1_n)}{d\ n^2}\right|_{n=\frac{1}{\ln(\alpha s)}-1} = \frac{(p\alpha s)^n(((n+1)n\ln(\alpha s))^2-2(n+1)\ln(\alpha s)+2)}{n^3} > 0$, which yields $\breve{n}$ given by (4).

Under the conditions of this proposition, employment in the firm is equal to $|N_{n^*}| = \Sigma_{k=0}^{n^*} s^k = (s^{n^*+1} - 1)/(s-1)$ with $n^* = n(w)$.

As mentioned above, the condition $\alpha \geq \frac{1}{2}e^{\frac{1}{2}}$ can be weakened to $\alpha s \geq e^{\frac{1}{2}}$. If an optimal firm level exists but $\alpha s < 1.65$ then the owner chooses the deepest organization $n^* = n(w)$ only if at that level the profit exceeds the profit of a one-level firm. Otherwise the flattest organization structure $n^* = 1$ is chosen. Finally, the firm is inactive if there does not exist an optimal firm level.

Example 4.3 We first consider an example where the values for the parameters are as suggested by Williamson (1967). He argues that the normal range for s is between 5 and 10. Now, let $s = 6,\ p = 1$, and $\alpha = 0.9$. In Table 1 we give some values for $\widetilde{\varphi}_0(f_n^1)$ and $\widetilde{\varphi}_n(f_n^1)$.

n	$\widetilde{\varphi}_0(f_n^1)$	$\widetilde{\varphi}_n(f_n^1)$
1	2.7	0.450
2	9.7	0.270
3	39.37	0.182
4	170.061	0.131
5	765.275	0.098

Table 1: $\rho = 1,\ s = 6,\ \alpha = 0.9,\ p = 1$

Since $\breve{n} < 1$, profit is increasing from level 1 onwards, and thus the critical level $n^c(p)$ equals 1. From the table it follows that if, for example, $w = \pi = 0.15$ then the critical level $n^c(0.15, 1)$ equals 1, and the optimal firm level is equal to 3, and the workers are pushed to their reservation wages. □

Example 4.4 Next we give an example with agency cost so high that $\alpha s < 1.65$. Let $s = 2,\ p = 1$ and $\alpha = 0.7$. Table 2 gives some values for $\widetilde{\varphi}_0(f_n^1)$ and $\widetilde{\varphi}_n(f_n^1)$.

n	$\widetilde{\varphi}_0(f_n^1)$	$\widetilde{\varphi}_n(f_n^1)$
1	0.700	0.350
2	0.653	0.163
3	0.686	0.086
4	0.768	0.048
5	0.896	0.028

Table 2: $\rho = 1,\ s = 2,\ \alpha = 0.7,\ p = 1$

Thus, the profit of the firm is minimal for $\breve{n} = 2$. The critical level equals 4. If, for example, $w = \pi = 0.15$ then the workers will only accept a position in a firm with level $n \leq 2$. In that case the owners will form the flattest hierarchical structure, and thus the firm will have one level.

If the reservation wage is low enough, for example, $w = 0.03$, then the owners of the firm will push the workers to their reservation wages and set firm level equal to 4. □

4.2.2 Cobb-Douglas outcome functions: full complementarity

Next we consider the case of a Cobb-Douglas production technology in which front-workers are homogeneous, but each worker is indispensable in the group. The Cobb-Douglas outcome function $f_n^0 \colon \{1, \ldots, s^n\} \to \mathbb{R}_+$ is given by $f_n^0(k) = s^n 1^k 0^{s^n - k}$ which equals s^n if $k = s^n$, and equals 0 otherwise. (Note that we normalized production such that the fully employed firm produces $f_n^0(s^n) = s^n$, and thus average labor productivity is the same as in the linear production technology of 4.2.1). In this case value added is given by

$$v^{f_n^0}(E) = \begin{cases} p(\alpha s)^n & \text{if } E \supset W_n \\ 0 & \text{else,} \end{cases}$$

i.e., if all positions in the n-level firm are occupied then value added equals $p(\alpha s)^n$, while value added equals zero if at least one position is not occupied. This reflects the indispensability of the working labor inputs. In this case the only non-zero dividend is that of the set of all workers and equals $\Delta^{v^{f_n^0}}(W_n) = p(\alpha s)^n$. According to the position pay-system, profit and wages are equal and are given by

$$\widetilde{\varphi}_0(f_n^0) = \widetilde{\varphi}_n(f_n^0) = \frac{p(\alpha s)^n}{\sum_{k=0}^n s^k} = \frac{p(\alpha s)^n (s-1)}{(s^{n+1} - 1)}. \tag{5}$$

For fixed α and n the wage of the workers is higher as compared to the linear production case. Again assume for simplicity that the level n can be any non-negative real number $n \in \mathbb{R}_+$. Then, profit and wages are decreasing in the number of hierarchical levels[16]. It follows that the owner sets firm level n not higher than 1, the flattest possible structure. The workers accept the positions in the firm if and only if $\frac{p\alpha s(s-1)}{s^2-1} \geq w$. In this case it are the owners of the firm who determine the firm level $n = 1$. For the individual workers who occupy a position in the firm this is the best possible structure.

Proposition 4.5 (Cobb-Douglas outcome function) *Consider a firm with Cobb-Douglas production technology and the position pay-system. If an optimal firm level exists, then the owner will choose the* **flattest** *organizational structure at level $n^* = 1$.*

In this case of full complementarity between front-workers, employment in the firm is equal to $|N_{n^*}| = s + 1$.

Example 4.6 Let $p = 1$, $\alpha = 0.7$, and $s = 2$. In Table 3 we give some values for $\widetilde{\varphi}_0(f_n^0)$ and $\widetilde{\varphi}_n(f_n^0)$. Clearly, profit is decreasing in firm level, and a finite critical level does not exist. □

[16]Since $\ln(\alpha) < 0$ (i.e., $\alpha < 1$) and $\ln(\alpha s) > 0$ (i.e., $\alpha s > 1$), it holds that $\frac{d\, \widetilde{\varphi}_0(f_n^0)}{d\, n} = \frac{p(\alpha s)^n (s-1)(s^{n+1}\ln(\alpha) - \ln(\alpha s))}{(s^{n+1}-1)^2} < 0$.

n	$\widetilde{\varphi}_0(f_n^0) = \widetilde{\varphi}_n(f_n^0)$
1	0.467
2	0.280
3	0.183
4	0.124
5	0.085
.	
∞	0.000

Table 3: $\rho \to 0,\ s = 2,\ \alpha = 0.7,\ p = 1$

4.2.3 The effect of a change in technology on a corporate firm

We have analyzed two types of corporate firms, both adapting the position pay-system and both endowed with a constant elasticity of substitution (CES) production technology: the linear and Cobb-Douglas production technologies. These are two extreme benchmark cases with perfectly substitutable labor inputs in the linear production technology and indispensable labor inputs in the Cobb-Douglas technology. We have seen that the difference between profit and worker wages in the linear technology firm is higher than in the Cobb-Douglas firm (where they are equal). As a consequence, the Cobb-Douglas firm will have at most one level while in the linear firm, for reasonable values of the efficiency parameter α, the firm will have its deepest firm level restricted by the reservation wage of workers. In a more general setting we could consider a firm with a homogeneous CES production outcome function $f_n^\rho\colon \{1, \dots, s^n\} \to \mathbb{R}_+$ given by[17] $f_n^\rho(k) = \gamma^{f_n^\rho}(k)^{\frac{1}{\rho}},\ 0 < \rho \leq 1$. Note that these are supermodular production functions[18]. Value added is given by $v^{f_n^\rho}(E) = p\alpha^n f_n^\rho(|E|)$ for $E \subset W_n$. The linear production technology corresponds to $\rho = 1$ and $\gamma^{f_n^1} = 1$, whereas the Cobb-Douglas technology corresponds to $\rho \to 0$ and $\gamma^{f_n^0} = s^n$. In such a CES firm[19] with complementarity parameter $\rho \in [0, 1]$, the optimal firm level is either one (as in the Cobb-Douglas case) or is determined by the reservation wage level (as in the linear case).

Formally, the shift from a deep to a flat organization depends on the front-workers complementarity parameter ρ. For an intermediate degree of complementarity, the profit decreases with n to a minimum, attained at $\breve{n}$, and then increases. The value of $\breve{n}$ (and thus the critical level $n^c(w, p)$) increases with complementarity (i.e. decreases with ρ). In the extreme case of

[17] A heterogeneous outcome function is a CES production function with m inputs if it is given by $f(x) = \gamma\left(\sum_{i=1}^m (x_i)^\rho\right)^{\frac{1}{\rho}},\ x \in \mathbb{R}^m,\ \gamma \in \mathbb{R},\ \rho \in (-\infty, 1]$.

[18] Since we assume the outcome function to be monotone ρ should be positive.

[19] Here we use the scale parameter $\gamma^{f_n^\rho}$ as a normalization factor in order to keep labor productivity per worker constant for different number of levels n, (and thus assuming labor productivity per worker to be independent of firm size). These firms satisfy the conditions of Proposition 3.2, and thus have a finite optimal firm level. An advantage of constant labor productivity is that the optimal firm level is not influenced by changes in labor productivity, but is determined by the distribution of the value added among the owner (as profit) and the employee positions (as positional wages).

a Cobb-Douglas production technology this minimum is infinite. Moreover, for fixed α and n the position-wage of a worker lies between the low position-wage in the linear production firm with a deep organization, and the high position-wage in the Cobb-Douglas firm with a flat organization[20].

This might explain the differences we observe in positional wages and firm levels. Although firms don't produce according to one particular homogeneous CES production technology, we do observe that workers whose positions have a high degree of complementarity usually get higher wages. For example, one of the front-positions in a hospital organization are the surgeon positions. The generation of value by a hospital strongly depends on the tasks assigned to the surgeon positions. Therefore we see that surgeon salaries are usually rather high compared to the salaries of hospital top management. On the other hand, the quality of the service provided by a desk office in, for example, a bank depends on the number and quality of the front desk-workers. But even if one front-desk position falls out, still a decent service can be delivered. In those organizations we usually see that top management salaries are much higher than front-worker wages[21]. The model developed here explains these differences, and consequently the difference in organizations. For example, there are usually not so many firm levels between a surgeon and hospital management.

We also can explain a shift in the technology of a given corporate firm. Consider the army. Its technology was clearly linear in the past, with a deep organizational structure. The technological innovations have shifted the technology to a Cobb-Douglas type, changing the army to a flatter organization with more equal and higher wages.

Not only the degree of substitutability of front-workers, but also the value of the parameter α determines the value of $\breve{n}$ (and thus of $n^c(w,p)$). A low value of α (high agency costs) yields a high value of $\breve{n}$, and thus there is a wide range of production technologies for which firm level is equal to 1, overruling the influence of the reservation wage of the workers on the firm level. If α increases, i.e., agency costs decrease, then the critical value $\breve{n}$ decreases, giving the owners the opportunity to push the workers to their reservation wage. In that case firm level, and thus employment, is influenced by the reservation wage of the workers.

Example 4.7 As an illustration of an intermediate case, consider $\rho = \frac{1}{2}$. Then the outcome function is given by $f_n^{1/2}(k) = \gamma^{f_n^{1/2}} k^2, k \in \{0, \ldots, s^n\}$. Since $f_n^{1/2}(s^n) = \gamma^{f_n^{1/2}} s^{2n}$, to keep labor productivity equal to one, the normalization factor must be $\gamma^{f_n^{1/2}} = \frac{1}{s^n}$. The value added then is given by

$$v^{f_n^{1/2}}(E) = p\left(\frac{\alpha}{s}\right)^n |E|^2, \text{ for all } E \subset W_n.$$

[20] So, for high enough substitutability of the labor inputs the optimal firm level is determined by the reservation wage level. Increasing complementarity will continuously decrease this reservation wage level up to a certain degree of complementarity, after which a further increase in complementarity makes optimal firm level discontinuously 'jump' to 1.

[21] Of course, also other features, such as the market or the quality and motivation of the workers, play a role in the determination of wages, but the role of complementarity of labor is often neglected in the literature.

Let $s = 2$, $p = 1$, and $\alpha = 0.7$. In Table 4 we give some values for $\widetilde{\varphi}_0(f_n^{1/2})$ and $\widetilde{\varphi}_n(f_n^{1/2})$. (Compare to Table 2 for the linear production technology and Table 3 for the Cobb-Douglas technology.)

n	$\widetilde{\varphi}_0(f_n^{1/2})$	$\widetilde{\varphi}_i(f_n^{1/2})$
1	0.583	0.408
2	0.482	0.200
3	0.465	0.105
4	0.490	0.058
5	0.548	0.033
6	0.637	0.020
7	0.762	0.010

Table 4: $\rho = \frac{1}{2}$, $s = 2$, $\alpha = 0.7$, $p = 1$

Thus, the profit of the firm is minimal for $n = 3$. The critical level equals 6. If $w = \pi = 0.15$ then the workers will only accept a position in a firm with level $n \leq 2$. In that case the owners will set firm level equal to 1 if the resevation profit π allows to do so. If, for example, $w = 0.015$ then the owner of the firm will push the workers to their reservation wages and set firm level equal to 6. Compared to Table 2 we see that the minimal profit is reached for a higher value of n. $\square$

5 Demand and supply in a corporate market economy

We finally define a market economy in which supply of consumer goods and demand for labor is set by a representative corporate firm as defined in this paper. Consider the triple $E = (F, \mathcal{C}, \mathcal{M})$, with a representative corporate firm F, a finite set of consumers $\mathcal{C}$ and a finite set of competitive markets $\mathcal{M}$. Such a triple E is called a *corporate market economy.*

The *internal equilibrium* of the corporate firm is determined by the optimal level of its organization, which is a function $n: \mathbb{R}^3_+ \to \mathbb{R}$, where $n(w, \pi, p)$ is the optimal firm level as given in Definition 3.1 for the wage w, the rate of return on capital π and the output price p. Here, the reservation prices in the previous sections are replaced by market prices, i.e. the reservation wage is determined by the market wage, the reservation profit is determined by the rate of return on capital and the output price is determined by the market price of the consumption good.

From Section 2 we know that the demand for labor, d_l, and the supply of commodities, s_c, are functions of market prices, $d_l(w, \pi, p) = \frac{s^{n(w,\pi,p)+1} - 1}{s-1} - 1$, and $s_c(w, \pi, p) = (\alpha s)^{n(w,\pi,p)}$ for any triple of market prices w, π and p. The capital needed for each active firm is equal to 1. Market demand and supply is determined by assuming that all firms are identical. So the firm is a representative firm in the industry, which may consist of more than one firm.

The number of firms is set equal to the capital demanded on the market. Assuming market supply of capital to be inelastic and given by $\overline{k} \in \mathbb{N}$, the number of firms on the market is then equal to $m = \overline{k}$. Assume that the rate of return on capital π is small enough to have no impact on the decision by the owner to activate the firm. Then only the relative prices between p and w matter.

Given an arbitrary pair of prices p and w we can determine the optimal size $n(w,p)$ of the firm by maximizing positional return on capital (with value added evaluated at price p) under the worker-participation constraint (determined by wage w) as done before. Market supply of the consumption good and market demand for labor then are given by $S_c(w,p) = ms^{n(w,p)}$, respectively, $D_l(w,p) = m\left(\frac{s^{(n(w,p)+1)}}{s-1} - 1\right)$.

The consumer side of the market consists of a set of consumers $\mathcal{C}$, each consumer $i \in \mathcal{C}$ having an initial endowment $\overline{c}_i \in \mathbb{R}_+$ of the consumption good, $\overline{l}_i \in \mathbb{R}_+$ units of time to spend as leisure or labor supply in a firm, and preferences over leisure and the consumption good represented by a utility function $u^i\colon \mathbb{R}^2_+ \to \mathbb{R}$. Assuming consumer preferences to satisfy the standard regularity conditions, this yields market demand for the consumption good and market supply of labor as functions of p and w. That determines consumer demand and supply on markets, $D_c(w,p)$ and $S_l(w,p)$.

Confronting market supply and market demand at given prices p and w, there may exist equilibrium prices p^* and w^*, defined as follows.

Definition 5.1 *A* **corporate equilibrium** *in a corporate market economy E is a pair of prices (w,p) and a firm level n, such that:*

1. *The firm level n is the optimal firm level given prices (w,p), and*
2. *The prices p,w are competitive equilibrium prices at which market supply equals market demand given firm level n.*

Given the standard regularity assumptions on the set of consumers, a corporate market equilibrium exists if n is continuous in prices[22]. For a firm with a linear production technology and $\alpha s \geq e^{\frac{1}{2}}$ a corporate market equilibrium thus exists since optimal firm level n is continuous in p and w, see Section 4.2.1. If positional return on capital is positive at the equilibrium output price and $\varphi_n(f_n)$ exceeds the equilibrium wage at $n = 1$, then these equilibrium prices are also corporate market equilibrium prices, and the optimal firm level is the firm level in the corporate market equilibrium. Otherwise, the firm level in a corporate market equilibrium equals $n = 0$, and its prices are too low to activate the workers.

Determining equilibrium prices in case the production technology is a Cobb-Douglas technology with complementary labor inputs we only have to consider firm level $n = 1$ since that

[22]Since high reservation wages will only support small firm level it holds that $n(w,p) \to 0$ if $\frac{w}{p} \to \infty$. On the other hand, low reservation wages result in large firm level since positional return on capital is increasing in n for large enough n, and thus $n(w,p) \to \infty$ if $\frac{w}{p} \to 0$.

is the optimal size for an active firm for all prices, see Section 4.2.2. In that case it is obvious that n is continuous in p and w. If the participation constraints can be satisfied at $n = 1$, then the equilibrium prices also are corporate market equilibrium prices, and $n = 1$ is the corporate market equilibrium firm level. Otherwise, corporate market equilibrium firm level equals $n = 0$, with corporate market equilibrium prices such that the workers do not want to activate the firm.

Proposition 5.2 *In a corporate market economy $E = (F, \mathcal{C}, \mathcal{M})$, where the technology of the corporate firm is either a linear outcome function (with $\alpha s \geq e^{\frac{1}{2}}$), or a Cobb-Douglas outcome function with non-increasing average labor productivity, the optimal firm level n^* is continuous in prices p and w.*

As a consequence of this proposition we have that in a corporate market economy a corporate market equilibrium exists if the firm is as described in Proposition 5.2. For intermediate cases of workers complementarity, however, corporate market equilibrium prices need not exist since optimal firm level n is discontinuous in w. This discontinuity occurs if positional return on capital (profit) of a firm with maximal level that can be supported by the equilibrium wage is lower than the positional return on capital of a firm with level equal to one. In the left picture of Figure 5 this is illustrated by a possible jump in firm level from 1 to $n^c(p)$.

The corporate market equilibrium concept generalizes upon the competitive equilibrium concept if one assumes that the economy becomes transparent on the long run. In that case the positional rents of cooperation within firms disappear and an overall Pareto efficiency is obtained.

6 Related literature and concluding remarks

In this paper we presented a model which endogenously determines the optimal number of echelons (called level) of a hierarchically structured firm with constant span of control. This hybrid model has cooperative as well as noncooperative features. One of the main novelties is the use of a cooperative pay-system which determines positional wages that may be higher than reservation wages. This phenomenon also appears in *efficiency wage theory* as discussed by, e.g., Stiglitz (1976), Akerlof (1984) and Yellen (1984). According to that theory laborers should be paid a rent on their equilibrium wage in order to stimulate them to put full effort in production and prevent them from shirking. A laborer losing its job or position at equilibrium wages can easily find a new position on the same conditions. However, paying rents on equilibrium wages makes it more difficult for laborers to find a new position on the same conditions, viz., including the same positional rents. Moreover, paying rents on equilibrium wages can induce unemployment which makes it even more difficult to find a new position with this positional rent. In our model wages can be higher than the reservation wages of workers. The *positional rent* of a position, i.e., the difference between the wage of a position and its reservation wage, can be compared to the rent of the efficiency wage.

Our model falls within new institutional economics which acknowledges that the neoclassical model of a firm, although very useful, is not sufficient to fully understand what happens inside and outside firms. We agree with Furutbotn (2001) that under bounded rationality and transaction costs profit maximization in the neoclassical way is not possible and other criteria have to be considered. In our model we assume a pay-system that assigns wages to the different positions in the firm to be given, since bounded rationality or transaction costs might prevent the employees to fully know their outside opportunities or have to make high costs in switching jobs or renegotiating every time an improvement can be made. Obviously, bounded rationality influences collective wage agreements and the resulting wages.

Our model also is in line with Rajan and Zingales (1998), who focus on the *control of access to a productive asset*. This in contrast with the literature on *incomplete contracts* which tries to explain the distribution of residual rights concerning the control over non-contractable assets (see, e.g., Grossman and Hart (1986), Hart and Moore (1990, 1999), and Maskin and Tirole (1999)), and thus puts *ownership* of assets central. Similar as in our paper, Rajan and Zingales (2001) try to explain firm formation focussing on the effects of agency costs and benefits on the firm's organization. They address the entrepreneurial problem of enlisting the cooperation of many agents necessary for production without ceding to them too much of the surplus generated by the enterprize. The tradeoff for the entrepreneur exists in designing an organization that gives subordinated managers access to the critical resource owned by the entrepreneur, without being expropriated by these managers. In our model, the key is the internal pay-system which is based on a system of bargaining from bottom up. The relation between asset ownership and relational contracts is studied in Baker, Gibbons and Murphy (2002).

Assuming that production processes in innovative industries usually use more complementary labor, we showed that such an increase in complementarity leads to flatter hierarchies. This is in line with Teece (1996) who studies the relation between firm structure and innovation and concludes that firms strongly depending on innovation have correspondingly flatter hierarchies.

Similar as in, e.g. DeCanio and Watkins (1998) and Garicano (2000), we neglect incentive problems and assume that if an employee is active in a firm then he or she is 'fully' productive. As Beggs (2001) argues, in order to get more insight in the functioning of hierarchical organizations it is best to focus on one of many aspects. In this sense these models are complementary to the models which focus on incentive problems such as Qian (1994) who endogenously determines the number of hierarchical echelons, the span of control and the wage scales by using optimal control techniques, and in that way extends the seminal work of Keren and Levhari (1979, 1983). Wage differences in our model arise not because of incentives, but because of differences in the production technology, which is not the case in

Qian (1994) who considers only one technology.

Since in our model of cooperative team production there is no uncertainty, issues like information problems and moral hazard do not arise. In this sense our model is complementary to the literature on *principal-agent* theory and *moral hazard* (see, e.g., Holmström (1979), Grossman and Hart (1983), Kessler (2000)). We can extend our model, for example by introducing risk as done in Prescott and Townsend (2002) who study how risk sharing can be a reason to form collective organizations by using principal-agent relations between these organizations and outsiders. These models mainly use non-cooperative game theory to study economic organizations in which binding agreements are not possible. Cooperative game theory yields valuable tools when studying cooperative situations in which binding agreements are possible. Recently, the use of cooperative game theory in analyzing economic organizations gained attention. Rotschild (2001) uses a solution similar to the Shapley value to determine the allocation of benefits among firms that form a cartel. Other applications can be found in, e.g. Curiel (1988), Hamers (1995) who study sequencing situations, Maniquet (2003) who studies queueing situations[23], Ambec and Sprumont (2002) who study water distribution problems, and Graham, Marshall and Richard (1990) who study cooperation among bidders in (English open bid and Second price sealed bid) auctions.

In the desciption of the firm we took the same approach as in Maskin, Qian and Xu (2000) who state that an organization is a "hierarchy of managers built on top of technology" where the technology is present in productive plants. However, since we assumed a constant span of control, and in the organization structure of the firm only vary the number of echelons, we do not study the organizational form of a hierarchy as done by Maskin, Qian and Xu (2000) who compare an M-form (multidivisional form in which the organization goes along institutional lines) with a U-form (unitary form in which the organization goes along regional lines) with respect to their effectiveness in giving incentives to managers.

Beggs (2001) uses optimal control techniques to determine the optimal structure of hierarchies when workers differ in the range of tasks they can perform. He studies how the complexity of tasks influences organizational structure. He explains why many organizations have a hierarchical structure by the economies of skilled workers. Skilled workers can make decisions without consulting other workers, while unskilled workers need to ask (superior) more skilled workers for advice or approval. Garicano (2000) develops a similar model in which he uses specialization instead of differences in worker skills. He explains the formation of hierarchies by a trade off between communication versus knowledge acquisition costs. In a "knowledge-based hierarchy" easy problems are solved by lower (production) echelons, while more exceptional or harder problems need to be passed on to higher echelons. Similar as in our model, in his model the decision "who must learn what and whom each worker should ask when confronted with an unknown problem" is part of the organization.

[23]The difference between these two models being the fact that in a sequencing situation the players or jobs are already positioned in an initial order which is not the case for queueing situations.

DeCanio and Watkins (1998) explain organizational structure from information processing by describing an organization as a 'pattern of information exchange among the agents', with an important role for the processing capabilities of the agents. Their conclusion that 'flattening of organizations would be one result of improved information processing capability of agents' is in line with our model since high complementarity of front-workers usually requires more skills than substitutable labor.

The friction between the internal cooperative behavior and the external competitive behavior is solved by accepting some imperfections in the labor markets and some hold up features in human resource management. Baron and Kreps (1999) observe that the Human Resource systems of successful firms, which systems are represented here by the firm's internal organization, often display practices reinforcing consistent themes or messages. Radically different Human Resource systems may exist because they face radically different external forces, but they also can flourish reasonably well in very similar situations, if only they are internally consistent. Consistency is obtained here because the organization is defined in terms of positions or jobs, and is directly adapted to the production technology. Human resource management aims at matching the productivity requirements of the position or the job with the productive capacities of the candidate to be employed. However, since the reward system is determined by the positional or job-productivity rather than by the individual's productivity outside this context, the productivity of some person depends crucially on the position in the firm's organization. Due to cooperation, the organization will enhance the productivity of that person drastically. Market wages on labor markets therefore refer to the potential match of some individual person with the job-productivity that may result in an organization, and are then assumed to correspond with that individual's productivity. Employment in the firm's organization creates firm-specific assets or human capital, which is controlled by the CEO. In the jargon of transaction cost economics, this feature of having all assets controlled by a single entity is called unified governance.

There are several directions for further research. In a homogeneous firm as considered in this paper the capacities of workers are identical. In a *heterogeneous* firm, workers and coordinators are not necessarily identical and their capacities and tasks may vary from very limited, such as routine work on a production line, to very sophisticated, such as work done by a surgeon in a hospital. Heterogeneous outcome functions then allow for heterogeneous organization structures, i.e., organization structures that do not necessarily have constant span of control, and even are not necessarily tree structures. Our approach can also be applied to such heterogeneous firms.

Secondly, the position pay-system as pay-system introduced here can be replaced by other systems. We already mentioned in Section 2 that the results obtained there hold for all pay-systems satisfying efficiency, vertical monotonicity and symmetry. Other examples of pay-systems satisfying these properties are the *egalitarian* pay-system (which assigns the same wage to each employee position which is equal to the profit assigned to the owner position), and every convex combination of this egalitarian pay-system and the position pay-system[24].

Finally, it is relevant to know how strong both in the short run and in the long run the internal cooperative forces are relative to the external competitive forces. The basic idea is that cooperation improves productivity sufficiently such that the firm can afford a labor cost above the reservation wage of labor at the lowest echelon. Or, said in another way, unschooled and unemployed labor can be made more productive by and employable in a firm with an adequate hierarchical internal organization. Further, labor in society is not homogeneous, but has personal capacities that are traded on differentiated labor markets. This heterogeneous labor interacts with the positions available in the internal organization of firms. From a dynamic point of view, labor input with a substitutable character that requires a deep firm organization in production, has incentives to transform in character by schooling, differentiation or specialization and to become suited for flat organizations. Concluding, we showed how technological change has an impact on the organization of firms on a micro-economic level. Through the impact on wages this influences employment on a macro-economic level, and therefore we showed how micro-economic forces should influence macro-economic policy making.

References

Akerlof, G. (1984), "Gift exchange and efficiency wage theory: four views", *American Economic Review Proceedings*, 74, 79-83.

Ambec, S. and Y. Sprumont (2002), "Sharing a River", *Journal of Economic Theory*, 107, 453-462.

Baker, G., R. Gibbons, and K.J. Murphy (2002), "Relational Contracts and the Theory of the Firm", *Quarterly Journal of Economics*, 117, 39-84.

Baron, J.N., and D.M. Kreps (1999), *Strategic Human Resources*, John Wiley & Sons, New York.

Beggs, A.W. (2001), "Queues and Hierarchies", *Review of Economic Studies*, 68, 297-322.

Borm, P., G. Owen, and S. Tijs (1992), "On the Position Value for Communication Situations", *SIAM Journal of Discrete Mathematics*, 5, 305-320.

Brink, R. van den (1996), "Skewness of the income distribution in a firm and the substitutability of labor inputs", *Research Memorandum* FEW 739, Tilburg University.

Brink, R. van den (1997), "An axiomatization of the disjunctive permission value for games with a permission structure", *International Journal of Game Theory*, 26, 27-43.

Brink, R. van den, and R.P. Gilles (1996), "Axiomatizations of the conjunctive permission value for games with permission structures", *Games and Economic Behavior*, 12, 113-126.

Coase, R.H (1937), "The Nature of the Firm," *Economica* 4, 386-405.

Coleman, J.S. (1980), "Authority systems", *Public Opinion Quarterly*, 44, 143-163.

[24]In the context of cooperative TU-games, convex combinations of egalitarian wage systems and Shapley value are considered in Joosten (1996) and Ju, Borm and Ruys (2004).

CURIEL, I. (1988), *Cooperative Game Theory and Applications*, Ph.D Thesis, Katholieke Universiteit Nijmegen.

DECANIO, S.J., AND W.E. WATKINS (1998), "Information Processing and Organizational Structure", *Journal of Economic Behavior and Organization*, 36, 275-294.

EVERETT, M.G., AND S.P. BORGATTI (1991), "Role coloring a graph", *Mathematical Social Sciences*, 26, 183-188.

FURUBOTN, E.G. (2001), "The New Institutional Economics and the Theory of the Firm", *Journal of Economic Behavior and Organization*, 45, 133-153.

GARICANO, L. (2000), "Hierarchies and the Organization of Knowledge in Production", *Journal of Political Economy* 108, 874–904.

GILLES, R.P., G. OWEN, AND R. VAN DEN BRINK (1992), "Games with permission structures: the conjunctive approach", *International Journal of Game Theory*, 20, 277-293.

GORDON, D.M. (1994), "Bosses of different stripes: a cross-national perspective on monitoring and supervision", *The American Economic Review*, AEA Papers and Proceedings, 375-379.

GRAHAM, D.A., R.C. MARSHALL, AND J.F. RICHARD (1990), "Differential Payments Within a Bidder Coalition and the Shapley Value", *American Economic Review* 80, 493-510.

GROSSMAN, S.J., AND O.D. HART (1983), "An Analysis of the Principal-Agent Problem," *Econometrica* 51, 7-45.

GROSSMAN, S.J., AND O.D. HART (1986), "The costs and benefits of ownership: a theory of vertical and lateral integration," *Journal of Political Economy* 94, 691-719.

HAMERS, H. (1995), *Sequencing and Delivery Situations: a Game Theoretic Approach*, Ph.D Thesis, University of Tilburg.

HARSANYI, J.C. (1959), "A bargaining model for cooperative n-person games," in *Contributions to the Theory of Games IV* (A.W. Tucker and R.D. Luce, Eds.), pp. 325-355. Princeton UP, Princeton.

HART, O., AND J. MOORE (1990), "Property rights and the nature of the firm," *Journal of Political Economy* 98, 1119-1158.

HART, O., AND J. MOORE (1999), "Foundations of incomplete contracts," *Review of Economic Studies* 66, 115-138.

HOLMSTRÖM, B. (1979), "Moral Hazard and Observability", *Bell Journal of Economics*, 10, 74-91.

HOFSTEDE, G. (1980), *Culture's Consequences. International Differences in Work-Related Values*, Sage Publications.

JOOSTEN, R. (1996), *Dynamics, Equilibria and Values*, Dissertation, Maastricht University.

JU, Y., P.E.M. BORM, AND P.H.M. RUYS (2004), "The consensus value : a new solution concept for cooperative games", Center Discussion paper 2004-50, Tilburg University, The Netherlands.

KEREN, M., AND D. LEVHARI (1979), "The optimum span of control in a pure hierarchy", *Management Science*, 25, 1162-1172.

KEREN, M., AND D. LEVHARI (1983), "The internal organization of the firm and the shape of average costs", *Bell Journal of Economics*, 14, 474-486.

KESSLER, A.S. (2000), "On Monitoring and Collusion in Hierarchies", *Journal of Economic Theory*, 91, 280-291.

MANIQUET, F. (2003), "A Characterization of the Shapley Value in Queueing Problems", *Journal of Economic Theory*, 109, 90-103.

MASKIN, E., Y. QIAN, AND C. XU (2000), "Incentives, Information and Organizational Form", *Review of Economic Studies*, 67, 359-378.

MASKIN, E., AND J. TIROLE (1999), "Unforeseen contingencies and incomplete contracts," *Review of Economic Studies*, 66, 83-114.

MILGROM P., AND J.ROBERTS (1994), "The economics of modern manufacturing: technology, strategy and organization", *American Economic Review*, 80, 511-528.

PEKEC, A., AND F.S. ROBERTS (2001), "The role assignment model nearly fits most social networks", *Mathematical Social Sciences*, 41, 275-293.

PRESCOTT, E.S., AND R.M. TOWNSEND (2002), "Collective Organizations versus Relative Performance Contracts: Inequality, Risk Sharing and Moral Hazard", *Journal of Economic Theory*, 103, 282-310.

QIAN, Y. (1994), "Incentives and loss of control in an optimal hierarchy", *The Review of Economic Studies*, 61, 527-544.

RADNER, R. (1992), "Hierarchy: the economics of managing", *Journal of Economic Literature*, 30, 1382-1415.

RAJAN, R.G., AND L. ZINGALES (1998), "Power in a Theory of the Firm", *Quarterly Journal of Economics*, 113, 387–432.

RAJAN, R.G., AND L. ZINGALES (2001), "The Firm as a Dedicated Hierarchy: A Theory of the Origins and Growth of Firms", *Quarterly Journal of Economics*, 116, 805-851.

ROSEN, S. (1982, "Authority, control, and the distribution of earnings", *Bell Journal of Economics, 13*, 311-323.

ROTHSCHILD, R. (2001), "On the Use of a Modified Shapley Value to Determine the Optimal Size of a Cartel", *Journal of Economic Behavior and Organization*, 45, 37-47.

SHAPLEY, L.S. (1953), "A value for n-person games", in *Annals of Mathematics Studies* 28 (Contributions to the Theory of Games Vol.2) (H.W. Kuhn and A.W. Tucker, Eds.), pp. 307-317. Princeton: Princeton UP.

SIMON, H. (1991), "Organizations and markets", *Journal of Economic Perspectives*, 5, 25-44.

STIGLITZ, J. (1976), "The efficiency wage hypothesis, surplus labour, and the distribution of income in L.D.C.'s", *Oxford Economic Papers, 28*, 185-207.

TEECE, D.J. (1996), "Firm Organization, Industrial Structure and Technological Innovation", *Journal of Economic Behavior and Organization*, 31, 193-224.

WILLIAMSON, O.E. (1967), "Hierarchical control and optimum firm level", *Journal of Political Economy*, 75, 123-138.

YELLEN, J.L. (1984), "Efficiency wage models of unemployment", *American Economic Review Proceedings*, 74, 200-205.